Summary

Much research on Full Waveform Inversion (FWI) focuses on advances in mathematical theory or on clever new algorithmic implementations. What is frequently omitted is a discussion of the engineering and workflow aspects that make FWI possible despite the many challenges provided by real data.

We find that contrary to the presumption of FWI as a black-box process, the typical FWI workflow requires data-specific processing steps and interaction with seismic interpretation data. This is particularly important in the critical first few iterations of FWI, and for continuous quality control throughout the FWI process. Examples of QC steps that benefit from interpretive processing are the analysis of the range of correlation frequencies and the generation of Rytov- or Born-approximation style differential data.

A second, often overlooked aspect is the use of detailed geologic knowledge to overcome intrinsic limitations of the inversion (such as edge effects, noise, or null-space-related problems) and to help stabilize the FWI convergence. We show how to partition volumetric data sets into separate compartments using interpreted horizons or geo-bodies. Inverted model parameter fields can then be conditioned with region-specific operators. Typical operations are min-max range clipping or conformal smoothing of velocity gradient volumes, often guided by geological input.

Introduction

Progress in computational capacity and data acquisition makes full waveform inversion a viable option for large-scale 3D high resolution velocity inversion. Over the last few years, several impressive FWI results have been presented for marine data (for example Sirgue et al. 2011) and land data (Plessix et al. 2010), sparking interest in how to robustly apply this technology to a larger range of data sets.

FWI explores the time characteristics and wave-shapes of seismic data to compute optimized model parameters of the subsurface. FWI's foundations have been developed over many years, starting with the pioneering work by Lailly (1983), who introduced the method of adjoint state to the seismic community. This method originated from Optimal Control theory and allows the efficient determination of high-dimensional wave propagation gradients. Tarantola (1984) used Lailly's research to derive the nonlinear inversion equations for seismic reflection data in the acoustic approximation. Original work on FWI was performed in the time-domain, but others introduced inversion algorithms in the frequency domain (Pratt and Worthington, 1990), Laplace domain (Shin and Ha, 2008) and hybrid domains (Liu et al. 1995). No matter the domain of application, FWI's most distinguishing feature as compared to Migration Velocity Analysis (MVA) is the direct evaluation of the objective function in the data domain. As such, much work in any FWI implementation will be performed directly on seismic pre-stack traces and, as demonstrated below, benefits from the rich set of trace processing tools available to the seismic processor.

We also claim that input from seismic data processors can be complemented by insights provided by seismic interpreters or geologists. Inversion results will never be perfect due to the non-linearity of the problem, imperfections in seismic data, and pragmatic assumptions made throughout the process. Geologic knowledge can fill in these shortcomings and help distinguish between inversion noise and signal, or suggest testing of various geologic scenarios if the inversion results are ambiguous.

The FWI workflow

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<th>FWI workflow items:</th>
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<tr>
<td>1. Generate synthetic data from reference model</td>
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<td>Forward modeling of every source-receiver pair</td>
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<td>2. Intelligently produce “differential” data</td>
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<td>Make amplitude insensitive, focus on phase</td>
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<td>3. FWI back-projection of data difference</td>
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<td>Back-propagate differential data into model</td>
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<td>4. QC and constrain model update</td>
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<td>Remove artifacts based on geologic knowledge</td>
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<td>Blue: Expensive, but well defined technology</td>
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<td>Red: Fast, but important, requires skill/craft/P</td>
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Figure 1. One iteration of a FWI workflow. Steps 2 and 4 benefit from interpretive processing using region constraints.

A FWI project consists of dozens or hundreds of iterations of the workflow depicted in Figure 1. We can characterize each FWI iteration as a seismic processing
Interpretive FWI with region constraints

workflow composed of 4 principle steps. Steps 1 and 3 (shown in blue) are compute-intensive seismic modeling and back-projection steps that are required to compute a new model update for each iteration. While specific implementations may vary greatly, established and well-defined algorithms (often based on finite-differencing) are readily available for both steps. Typically, the algorithms in steps 1 and 3 are less dependent on the details of the FWI data and require little human intervention by the FWI processor. On the other hand, items 2 and 4, (shown in red) are often the crux in FWI processing and need to be carefully tailored towards project-specific parameters and data situations. If those steps are implemented in a modular and integrated environment, they can be flexibly tuned from project to project or even between FWI iteration as needed. Below, we focus on those computationally fast, but highly interpretive steps (Step 2 and 4).

Step 2: Produce FWI differential data

Inversion algorithms typically used in FWI require the computation of the gradient of the wavefield with respect to each model parameter. Considering the huge amount of parameters (such as velocity samples) of the model space, at first glance, this appears to be an impossible task. The contribution of Lailly (1983) has been to point out an efficient alternative often referred to as method of adjoint state. Instead of direct computation of a gradient, it is enough to back-propagate a data differential and correlate it with the forward modeling of a virtual source. This FWI data differential (observed-wavefield minus modeled-wavefield) plays a crucial role in the success of the inversion. It represents the part of the data that is not taken into account by the current model and, mathematically, can be expressed as

$$\Psi_{diff} = \Psi_{obs} - \Psi_{model}.$$

In practice, a straight algebraic subtraction of sample amplitudes of both wavefields (such as motivated by the Born approximation) is often found lacking and suffers from cycle-skipping. This situation occurs frequently during early iterations of FWI when both data sets are substantially different due to a bad starting velocity model or poor control of the source wavelet.

If the FWI workflow is flexible and seismic processing tools are readily available, we can investigate alternative ways to derive the differential wavefield. As shown in Figure 2 (for a SEAM model FWI) and Figure 3 (for FWI of the SEG/EAGE overthrust), we identify the range of frequencies that can be used to compare both datasets without cycle-skips. This can be performed as a function of spatial position and source-receiver offset by testing for phase errors (Figure 2), or by windowed correlation at selected CMP locations and offsets (Figure 3).

After identifying the frequency range and the subset of the shot gathers that can produce meaningful FWI inversion results, we investigate alternative ways of computing $\Psi_{diff}$. Calling $A_{obs}$, $A_{model}$, $\Phi_{obs}$, $\Phi_{model}$ the amplitude and phase spectra of the observed and modeled data, respectively, we can compute the differential phase as

$$\Delta \Phi = [\ln(A_{obs}) - \ln(A_{model})] + \{ \Phi_{obs} - \Phi_{model} \}.$$
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Under the Rytov-approximation, one can then derive the amplitude-insensitive differential wavefield as a function of phase difference

\[ \psi_{\text{diff}} = \psi_{\text{model}} [ \Phi_{\text{obs}} - \Phi_{\text{model}} ] e^{i \frac{\pi}{2}}. \]

This equation shows that the differential wavefield can be computed by the modeled wavefield, convolved with a 90-degree filter of amplitude spectrum \([ \Phi_{\text{obs}} - \Phi_{\text{model}} ]\). If this difference is linear in frequency, the differential wavefield is simply the model wavefield multiplied by a derivative filter.

This wavefield-phase based result indicates only one of many possible alternatives to computing \(\psi_{\text{diff}}\) by straight subtraction. It is a relevant example that demonstrates how a modular FWI processing setup allows performing ad-hoc data QC and application of data-motivated alternative methods during the FWI iterations.

Step 4: QC and constrain model updates

![Diagram of model update process](image)

After computing \(\psi_{\text{diff}}\), the differential data are back-propagated for a new estimation of the model gradient. An example raw velocity change (the result of iteration 6 in a FWI inversion of the SEG/EAGE overthrust model) is shown in the upper part of Figure 4. It is well known that inversion is a highly non-linear process prone to instabilities. So it is encouraging to observe that this particular velocity update is correlated with geologic markers such as the main fault plane and seismic horizons. Other features, such as the smearing in the left-lower corner can be attributed to inversion edge artifacts and removed. In other parts of the model, it is more challenging to distinguish between inversion noise and useful model updates.

![Diagram of constrained model update](image)

The project geologist or seismic interpreter is the most qualified to make educated guesses and, as depicted in the lower part of Figure 4, to propose how to constrain the model. In many situations, he may also discard the current model update or suggest testing of alternative geologic scenarios that are plausible for the project area.

In a two-dimensional model, applying such constraints may appear trivial. In three-dimensions, applying region-specific constraint algorithms can be daunting. In addition, each constraining scheme should be scalable in complexity and be able to incorporate independent geologic information.

Early FWI iterations may only need simple structural regions, such as a delineation of the water layer and shallow sediments. In later iterations, when deeper and finer details of the velocity model are resolved, a much more sophisticated structural framework consistent with carefully interpreted horizons, faults and well-based time-depth information is required.
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Figures 5 and 6 show examples of how seismic interpreter input can guide the constraining of models. Horizons (in early iterations) or geo-bodies can be used to define a structural model that is consistent with the available geologic information. Then, for each region inside the structural framework, we select constraint operations such as conformal smoothing, averaging or clipping from a portfolio of constraint algorithms.

We analyzed many of the publically available FWI results and in almost all cases believe that geologically-motivated, region-based constraint operation may potentially further improve the FWI results.

Figure 5. FWI Apply Region Constraints

Conclusions

Recent case studies have shown that FWI can produce spectacular results on industrial-scale real 3D data sets. FWI clearly provides a jump in imaging technology, but should not be considered as a single-step, monolithic algorithm. Our modular and interpretive view of FWI processing allows customization of individual parts of the workflow. This flexibility is important to respond to particular challenges of the given data sets and to fully exploit the skills of the FWI practitioner.

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EDITED REFERENCES
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